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A CONTINUOUS-DISCRETE APPROACH TO THE FREE VIBRATION ANALYSIS OF STIFFENED PIERCED WALLS ON FLEXIBLE FOUNDATIONS

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Abstract — A hybrid approach, based on the analysis of an equivalent continuous medium and a discrete lumped mass system, is presented in this paper for conducting dynamic analysis of stiffened coupled shear walls supported on foundations which are flexible rotationally, and in both the vertical and horizontal directions. The effect of shear deformation on the free vibration behaviour of coupled shear walls reinforced by stiffening beams located at the top and the base level, and at an arbitrary level along the structural height is considered. The stiffened coupled shear wall system is represented by either a continuous system or a discrete system at various stages of the analysis. The advantage of using these two ways of representing the shear wall system is that the stiffness and mass matrices required for structural dynamic analysis are easily obtained. Numerical studies on a typical example structure, carried out by this continuous-discrete approach, to investigate the effects of shear deformation and the incorporation of stiffening beams on the free vibration characteristics of stiffened coupled shear wall structures, are also presented.

NOTATION

A_1, A_2	cross-sectional area of walls 1 and 2
A	$A_1 + A_2$
A_b	cross-sectional area of connecting beams
A_m	cross-sectional area of stiffening beams
b	clear span of connecting beams
E	elastic modulus of walls and connecting beams
E_m	elastic modulus of stiffening beams
G	elastic shear modulus of walls and connecting beams
G_m	elastic shear modulus of stiffening beams
H	total height of walls
H_1	position of the intermediate stiffening beam
H_p	height of lateral unit force applied on walls
h	storey height
I_1, I_2	second moment of area of walls 1 and 2
I	$I_1 + I_2$
I_m	second moment of area of stiffening beams
I_b	second moment of area of connecting beams
K_h	horizontal foundation stiffness
K_r	rotational foundation stiffness
K_v	vertical foundation stiffness
l	distance between centroidal axes of walls
M_o	applied moment
Q	total shear force in walls
q	laminar shear in the equivalent medium
s	relative settlement of walls
S_m	relative flexural rigidity of stiffening beams
T	axial force in each wall
V_m	shear force in stiffening beams
x	height coordinate
y	total lateral deflection of walls

y_M lateral deflection of walls due to bending
y_Q lateral deflection of walls due to shear deformation
μ_b, μ_m, μ_w modifying factor for calculating shear deformation of connecting beams, stiffening beams and walls with average cross-sectional shear stress
θ_0 rotational base deformation
ρ density of walls
α, β, γ structural parameters

INTRODUCTION

As the height of buildings increases, it becomes more and more important to provide the structures of buildings with sufficient stiffness against lateral loads arising from wind or earthquakes. Reinforced concrete shear walls are recognized as one of the more efficient structural systems for such purposes (Irwin, 1984); however, such walls are very often weakened by vertical bands of openings which are required for doors, windows and corridors (Fig. 1). These walls with openings, called pierced or coupled shear walls, behave like a system consisting of two vertical parallel cantilevers coupled by intermittent cross or lintel beams with rigid joints throughout the structural height. Over the years, numerous studies have been carried out on the static analysis of coupled shear walls (Rosman, 1964; Coull and Choudhury, 1967; Tso and Chan, 1972), and a convenient summary of the principal methods of analysis has been provided by Taranath (1989). These studies have led to development of a more effective configuration, called stiffened coupled shear walls, in which the stiffness of the coupled walls is improved by stiff deep beams or belt trusses incorporated at various levels (Coull, 1974; Choo and Coull, 1984; Chan and Kuang, 1988, 1989), resulting in beneficial effects on the behaviour of the structural system.

In most practical situations, the width of the walls (say, of the order of 6–10 m) will be much larger than the depth of the cross beams connecting the walls [see Fig. 1(a)], since a typical storey height for buildings is only of the order of 3 m and the depths of the cross beams are less than the storey height. Consequently the bending moments fed into the walls from the connecting beams at each floor level tend to be relatively small compared with the wall moments over most of the height. This is the opposite of the situation with conventional rigidly connected building frames, where the beams play roles as important as columns for moment resistance. Based on this fact, an idea using a continuous medium with equivalent stiffness to replace the discrete connecting beams was proposed. This idea has led to the establishment of a simple yet accurate continuum method, with much less data preparation effort and computational costs involved compared with discrete methods (e.g. finite element method), for conducting the static analysis of this type of structure (Coull and Smith, 1967). However, employing the continuum method for the free vibration analysis of coupled shear walls, which is inevitably required for obtaining the natural frequencies and mode shapes to assess the loads induced by dynamic effects of wind or earthquakes and to perform related structural design calculations, results in a sixth-order differential equation for which

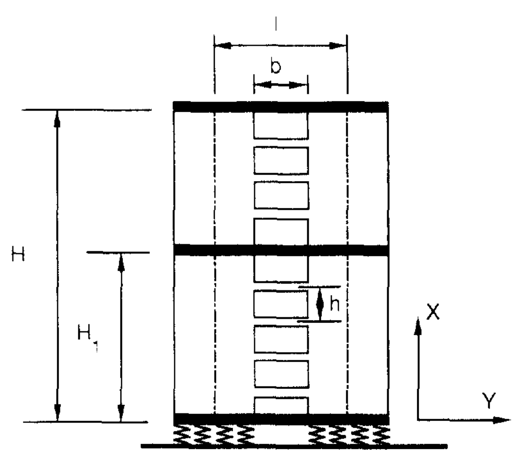

Fig. 1. Stiffened coupled shear walls on flexible foundation.

there is no closed solution. To overcome this mathematical difficulty, techniques based on Galerkin's method of weighted residuals and Ritz–Galerkin's method have been proposed for approximate solution of the equation (Coull and Mukherjee, 1973; Mukherjee and Coull, 1973, 1974). These techniques enable the eigenvalue equations for structural free vibrations to be reduced to a set of linear equations. Thus, the original complex differential equation is converted to a set of linear algebraic equations for which there are standard solutions. Nevertheless, it is very hard to estimate the inherent errors arising from the approximation in the continuous approach, especially when coupled shear walls are stiffened by stiff beams and are supported on flexible foundations. In addition, previous studies on the free vibrations of coupled shear wall structures by the continuous approach have been conducted without taking account of shear effects.

A continuous-discrete approach, based on the analysis of an equivalent continuum medium and a discrete lumped mass system, which overcomes the above-mentioned shortcomings whilst maintaining the advantages of the continuous approach, has been developed for the free vibration analysis of coupled shear walls (Li and Choo, 1994a–c). The basic idea of the hybrid approach is to represent the coupled shear walls as both a continuous system and a discrete system at different stages of dynamic analysis. By so doing, it is possible to obtain, relatively easily, the stiffness and mass matrices required for dynamic analysis. The accuracy and efficiency of the continuous-discrete approach have been verified against experimental data and classical methods of analysis.

The purpose of this paper is to extend the continuous-discrete approach to the free vibration analysis of stiffened coupled shear walls situated on flexible foundations, allowing for the effect of shear deformation. The stiffening beams may locate at the top and bottom of the structure and at an arbitrary intermediate level along the height of the walls. The effects of shear deformation and the reinforcement of stiffening beams on the free vibration characteristics of coupled shear wall systems are investigated.

OUTLINE OF METHODOLOGY

For dynamic analysis of structural systems, both the mass and stiffness matrices are required. In order to obtain these two matrices for a stiffened coupled shear wall structure supported on a deformable foundation, first consider the structural system as a discrete system as shown in Fig. 2(b). This is the multi-degree-of-freedom lumped mass system commonly used for dynamic analysis of structures in engineering. The mass matrix **M** of this discrete system can easily be obtained by

$$\mathbf{M} = \text{diag}\left[m_1, m_2, \ldots, m_n, m_{n+1}\right], \tag{1}$$

where m_i ($i = 1, 2, \ldots, n+1$) are the lumped masses at various locations along the height of the walls and n is the number of the lumped masses above the ground.

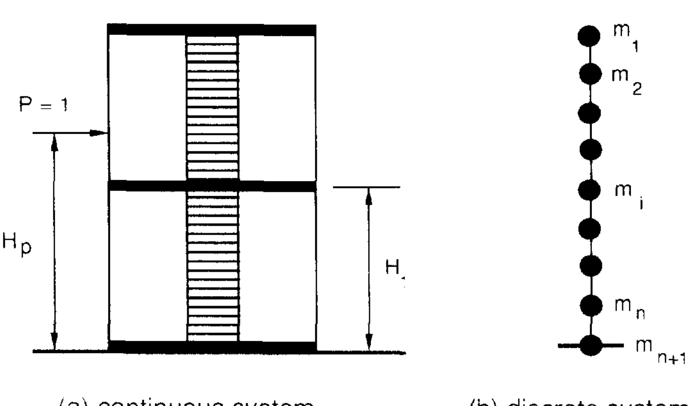

Fig. 2. Analytical systems.

The mass of coupled shear wall structures is mainly due to the walls: because the wall can be regarded as uniform throughout its height, it is rational and convenient to assume that the mass of the coupled shear wall structures may be represented by a number of lumped masses located evenly throughout the height of the walls. Thus the lumped mass m_i can be simply calculated by

$$m_i = \frac{1}{2n} M_1 \quad \text{for } i = 1 \text{ or } n+1 \tag{2a}$$

$$m_i = \frac{1}{n} M_1 \quad \text{for } 1 < i < n+1, \tag{2b}$$

where M_1 is the total mass of the structure.

The stiffness matrix **K** of the structure can be obtained by inverting the flexibility matrix **F**, i.e.

$$\mathbf{K} = \mathbf{F}^{-1} \tag{3}$$

The values of the ith column of elements in the flexibility matrix **F** for the structure considered represent the lateral deflections of the walls at all levels where lumped masses are located, induced by a unit force applied horizontally at the location of the ith lumped mass. For this purpose, consider the stiffened coupled shear walls as a continuous system [shown in Fig. 2(a)]. The entire flexibility matrix of the structure may be formed by repeating the static analysis for a unit lateral force on the walls at each and every level for which there is an assumed lumped mass.

Having obtained both the mass and stiffness matrices, the free vibration analysis of the stiffened coupled shear walls can be conducted by solving the following standard frequency equation and eigenvalue equation for multi-degree-of-freedom systems:

$$|\mathbf{K} - \omega^2 \mathbf{M}| = 0 \tag{4}$$

$$(\mathbf{K} - \omega^2 \mathbf{M})\mathbf{u} = \mathbf{0}, \tag{5}$$

where ω and **u** are the circular frequency and deflection vector of vibration, respectively, of the structure.

Obviously, the key aspect of the approach described above is the derivation of the lateral deflection expressions for elastically based stiffened coupled shear walls subjected to a unit horizontal force at an arbitrary level.

LATERAL DEFLECTION EXPRESSIONS

Consider an elastically based coupled structural wall system stiffened by a top and a bottom stiffening beam and an intermediate stiffening beam at the level $x = H_1$, as shown in Fig. 1. This system is subjected to a horizontal unit force at the position $x = H_p$ [Fig. 2(a)]. The connecting beams of the system are assumed to be replaced by continuous laminae with equivalent stiffness. It is further assumed that the centre-line of laminae passes through the points of contraflexure of the connecting and stiffening beams. If a "cut" is

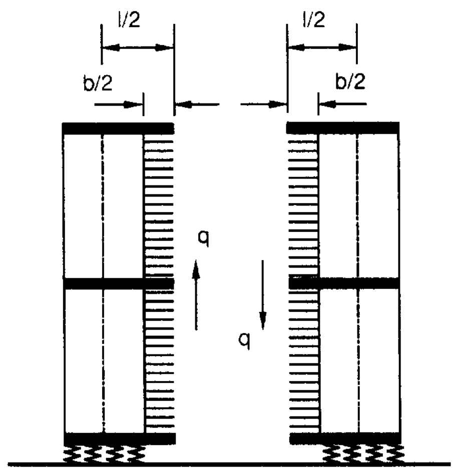

Fig. 3. Substitute structure.

made along the centre-line of laminae (Fig. 3), the relative vertical deflection of the cut ends of the laminae must be zero. This leads to the following compatibility equations:

if $H_1 \leqslant H_p$

$$l\frac{dy_{M2}}{dx}+\frac{Qb\mu_w}{G(A_1+A_2)}-\frac{hb^3q_2}{12EI_b}-\frac{b\mu_b hq_2}{GA_b}$$
$$-\frac{1}{E}\left(\frac{1}{A_1}+\frac{1}{A_2}\right)\left[\int_0^{H_1}T_0\,dx+\int_{H_1}^{H_p}T_1\,dx+\int_{H_p}^{x}T_2\,dx\right]-s=0 \quad \text{for } H_p<x\leqslant H \tag{6a}$$

$$l\frac{dy_{M1}}{dx}+\frac{Qb\mu_w}{G(A_1+A_2)}-\frac{hb^3q_1}{12EI_b}-\frac{b\mu_b hq_1}{GA_b}$$
$$-\frac{1}{E}\left(\frac{1}{A_1}+\frac{1}{A_2}\right)\left[\int_0^{H_1}T_0\,dx+\int_{H_1}^{x}T_1\,dx\right]-s=0 \quad \text{for } H_1<x\leqslant H_p \tag{6b}$$

$$l\frac{dy_{M0}}{dx}+\frac{Qb\mu_w}{G(A_1+A_2)}-\frac{hb^3q_0}{12EI_b}-\frac{b\mu_b hq_0}{GA_b}$$
$$-\frac{1}{E}\left(\frac{1}{A_1}+\frac{1}{A_2}\right)\int_0^{x}T_0\,dx-s=0 \quad \text{for } 0\leqslant x\leqslant H_1 \tag{6c}$$

and if $H_p < H_1$

$$l\frac{dy_{M2}}{dx}+\frac{Qb\mu_w}{G(A_1+A_2)}-\frac{hb^3q_2}{12EI_b}-\frac{b\mu_b hq_2}{GA_b}$$
$$-\frac{1}{E}\left(\frac{1}{A_1}+\frac{1}{A_2}\right)\left[\int_0^{H_p}T_0\,dx+\int_{H_p}^{H_1}T_1\,dx+\int_{H_1}^{x}T_2\,dx\right]-s=0 \quad \text{for } H_1<x\leqslant H \tag{7a}$$

$$l\frac{dy_{M1}}{dx}+\frac{Qb\mu_w}{G(A_1+A_2)}-\frac{hb^3q_1}{12EI_b}-\frac{b\mu_b hq_1}{GA_b}$$
$$-\frac{1}{E}\left(\frac{1}{A_1}+\frac{1}{A_2}\right)\left[\int_0^{H_p}T_0\,dx+\int_{H_p}^{x}T_1\,dx\right]-s=0 \quad \text{for } H_p<x\leqslant H_1 \tag{7b}$$

$$l\frac{\mathrm{d}y_{M0}}{\mathrm{d}x}+\frac{Qh\mu_{\mathrm{w}}}{G(A_1+A_2)}-\frac{hb^3q_0}{12EI_{\mathrm{b}}}-\frac{h\mu_{\mathrm{b}}hq_0}{GA_{\mathrm{b}}}$$
$$-\frac{1}{E}\left(\frac{1}{A_1}+\frac{1}{A_2}\right)\int_0^x T_0\,\mathrm{d}x-s=0\quad \text{for } 0\leqslant x\leqslant H_p, \tag{7c}$$

where y_{M0}, q_0, T_0; y_{M1}, q_1, T_1; and y_{M2}, q_2, T_2 are the lateral deflection of the walls, the laminar shear in the connecting beams and the axial force in the walls. The subscripts 0, 1 and 2 represent the region between the base stiffening beam and the lateral unit force or the intermediate stiffening beam; between the lateral unit force and the intermediate stiffening beam; and between the lateral unit force or the intermediate stiffening beam and the top stiffening beam, respectively. The successive terms represent the vertical deflection of the midpoint of the lamina due to the bending and shear deformation of the walls, bending and shear deformation of the laminae, axial deformation of the walls and the relative vertical foundation settlement.

The moment–curvature relationship for the walls is

$$EI\frac{\mathrm{d}^2y_M}{\mathrm{d}x^2}=M_e-Tl \tag{8}$$

(I is the sum of the second moments of area of walls 1 and 2), where the axial forces in each wall at different levels are given by:

if $H_1\leqslant H_p$

$$T_2=V_{\mathrm{mt}}+\int_x^H q_2\,\mathrm{d}x \qquad \text{for } H_p<x\leqslant H \tag{9a}$$

$$T_1=V_{\mathrm{mt}}+\int_{H_p}^H q_2\,\mathrm{d}x+\int_x^{H_p} q_1\,\mathrm{d}x \qquad \text{for } H_1<x\leqslant H_p \tag{9b}$$

$$T_0=V_{\mathrm{mt}}+\int_{H_p}^H q_2\,\mathrm{d}x+\int_{H_1}^{H_p} q_1\,\mathrm{d}x+V_{\mathrm{mi}}+\int_x^{H_1} q_0\,\mathrm{d}x \qquad \text{for } 0\leqslant x\leqslant H_1 \tag{9c}$$

and if $H_p<H_1$

$$T_2=V_{\mathrm{mt}}+\int_x^H q_2\,\mathrm{d}x \qquad \text{for } H_1<x\leqslant H \tag{10a}$$

$$T_1=V_{\mathrm{mt}}+\int_{H_1}^H q_2\,\mathrm{d}x+V_{\mathrm{mi}}+\int_x^{H_1} q_1\,\mathrm{d}x \qquad \text{for } H_p<x\leqslant H_1 \tag{10b}$$

$$T_0=V_{\mathrm{mt}}+\int_{H_1}^H q_2\,\mathrm{d}x+V_{\mathrm{mi}}+\int_{H_p}^{H_1} q_1\,\mathrm{d}x+\int_x^{H_p} q_0\,\mathrm{d}x \qquad \text{for } 0\leqslant x\leqslant H_p \tag{10c}$$

in which V_{mt} and V_{mi} are the shear forces in the top and the intermediate stiffening beams, respectively.

The applied bending moment M_e can be represented by

$$M_e=\begin{cases}H_p-x & x\leqslant H_p\\ 0 & x>H_p\end{cases}. \tag{11}$$

The total shear force in both walls Q can be expressed as

$$Q = \begin{cases} 1 & x \leqslant H_p \\ 0 & x > H_p \end{cases}. \tag{12}$$

The laminar shear flow intensity q can be obtained by differentiating the axial force in each wall, i.e.

$$q = -\frac{\mathrm{d}T}{\mathrm{d}x}. \tag{13}$$

Differentiating eqn (6) or (7) and combining eqns (8) and (13) to eliminate the variables y_M and q yields the governing equation for T

$$\frac{\mathrm{d}^2T}{\mathrm{d}x^2} - \alpha^2 T = -\gamma M_e, \tag{14}$$

where

$$\gamma = \frac{12 I_b l}{h b^3 I}\left(1 + \frac{12\mu_b E I_b}{G A_b b^2}\right)^{-1} \qquad \alpha^2 = \gamma\left(l + \frac{IA}{A_1 A_2 l}\right)$$

$(A = A_1 + A_2)$ for which the complete solutions are as follows:

if $H_1 \leqslant H_p$

$$T_2 = B_2 \cosh \alpha x + C_2 \sinh \alpha x \qquad \text{for } H_p < x \leqslant H \tag{15a}$$

$$T_1 = B_1 \cosh \alpha x + C_1 \sinh \alpha x + \frac{\gamma}{\alpha^2}(H_p - x) \qquad \text{for } H_1 < x \leqslant H_p \tag{15b}$$

$$T_0 = B_0 \cosh \alpha x + C_0 \sinh \alpha x + \frac{\gamma}{\alpha^2}(H_p - x) \qquad \text{for } 0 \leqslant x \leqslant H_1 \tag{15c}$$

and if $H_p < H_1$

$$T_2 = B_2 \cosh \alpha x + C_2 \sinh \alpha x \qquad \text{for } H_1 < x \leqslant H \tag{16a}$$

$$T_1 = B_1 \cosh \alpha x + C_1 \sinh \alpha x \qquad \text{for } H_p < x \leqslant H_1 \tag{16b}$$

$$T_0 = B_0 \cosh \alpha x + C_0 \sinh \alpha x + \frac{\gamma}{\alpha^2}(H_p - x) \qquad \text{for } 0 \leqslant x \leqslant H_p, \tag{16c}$$

where B_2, C_2, B_1, C_1, B_0 and C_0 are integration constants.

The corresponding expressions for the laminar shear, derived by using eqn (13), are given by

if $H_1 \leqslant H_p$

$$q_2 = -(B_2 \alpha \sinh \alpha x + C_2 \alpha \cosh \alpha x) \qquad \text{for } H_p < x \leqslant H \tag{17a}$$

$$q_1 = -\left(B_1 \alpha \sinh \alpha x + C_1 \alpha \cosh \alpha x - \frac{\gamma}{\alpha^2}\right) \qquad \text{for } H_1 < x \leqslant H_p \tag{17b}$$

$$q_0 = -\left(B_0 \alpha \sinh \alpha x + C_0 \alpha \cosh \alpha x - \frac{\gamma}{\alpha^2}\right) \qquad \text{for } 0 \leqslant x \leqslant H_1 \tag{17c}$$

and if $H_p < H_1$

$$q_2 = -(B_2\alpha \sinh \alpha x + C_2 \alpha \cosh \alpha x) \qquad \text{for } H_1 < x \leqslant H \tag{18a}$$

$$q_1 = -(B_1\alpha \sinh \alpha x + C_1 \alpha \cosh \alpha x) \qquad \text{for } H_p < x \leqslant H_1 \tag{18b}$$

$$q_0 = -\left(B_0\alpha \sinh \alpha x + C_0 \alpha \cosh \alpha x - \frac{\gamma}{\alpha^2}\right) \qquad \text{for } 0 \leqslant x \leqslant H_p. \tag{18c}$$

To determine the shear forces in the stiffening beams, consider the following compatibility requirements at the points of contraflexure:

if $H_1 \leqslant H_p$

$$l\frac{\mathrm{d}y_{M2}}{\mathrm{d}x}\bigg|_{x=H} - \frac{V_{\mathrm{mt}}b^3}{12E_{\mathrm{mt}}I_{\mathrm{mt}}} - \frac{b\mu_{\mathrm{mt}}V_{\mathrm{mt}}}{G_{\mathrm{mt}}A_{\mathrm{mt}}} - \frac{1}{E}\left(\frac{1}{A_1}+\frac{1}{A_2}\right)\left[\int_0^{H_1} T_0\,\mathrm{d}x + \int_{H_1}^{H_p} T_1\,\mathrm{d}x + \int_{H_p}^{H} T_2\,\mathrm{d}x\right] - s = 0 \tag{19a}$$

$$l\frac{\mathrm{d}y_{M1}}{\mathrm{d}x}\bigg|_{x=H_1} - \frac{V_{\mathrm{mi}}b^3}{12E_{\mathrm{mi}}I_{\mathrm{mi}}} - \frac{b\mu_{\mathrm{mi}}V_{\mathrm{mi}}}{G_{\mathrm{mi}}A_{\mathrm{mi}}} - \frac{1}{E}\left(\frac{1}{A_1}+\frac{1}{A_2}\right)\int_0^{H_1} T_0\,\mathrm{d}x - s = 0 \tag{19b}$$

$$l\frac{\mathrm{d}y_{M0}}{\mathrm{d}x}\bigg|_{x=0} - \frac{V_{\mathrm{mb}}b^3}{12E_{\mathrm{mb}}I_{\mathrm{mb}}} - \frac{b\mu_{\mathrm{mb}}V_{\mathrm{mb}}}{G_{\mathrm{mb}}A_{\mathrm{mb}}} - s = 0 \tag{19c}$$

and if $H_p < H_1$

$$l\frac{\mathrm{d}y_{M2}}{\mathrm{d}x}\bigg|_{x=H} - \frac{V_{\mathrm{mt}}b^3}{12E_{\mathrm{mt}}I_{\mathrm{mt}}} - \frac{b\mu_{\mathrm{mt}}V_{\mathrm{mt}}}{G_{\mathrm{mt}}A_{\mathrm{mt}}} - \frac{1}{E}\left(\frac{1}{A_1}+\frac{1}{A_2}\right)\left[\int_0^{H_p} T_0\,\mathrm{d}x + \int_{H_p}^{H_1} T_1\,\mathrm{d}x + \int_{H_1}^{H} T_2\,\mathrm{d}x\right] - s = 0 \tag{20a}$$

$$l\frac{\mathrm{d}y_{M1}}{\mathrm{d}x}\bigg|_{x=H_1} - \frac{V_{\mathrm{mi}}b^3}{12E_{\mathrm{mi}}I_{\mathrm{mi}}} - \frac{b\mu_{\mathrm{mi}}V_{\mathrm{mi}}}{G_{\mathrm{mi}}A_{\mathrm{mi}}} - \frac{1}{E}\left(\frac{1}{A_1}+\frac{1}{A_2}\right)\left[\int_0^{H_p} T_0\,\mathrm{d}x + \int_{H_p}^{H_1} T_1\,\mathrm{d}x\right] - s = 0 \tag{20b}$$

$$l\frac{\mathrm{d}y_{M0}}{\mathrm{d}x}\bigg|_{x=0} - \frac{V_{\mathrm{mb}}b^3}{12E_{\mathrm{mb}}I_{\mathrm{mb}}} - \frac{b\mu_{\mathrm{mb}}V_{\mathrm{mb}}}{G_{\mathrm{mb}}A_{\mathrm{mb}}} - s = 0, \tag{20c}$$

where μ_{mt}, E_{mt}, G_{mt}, I_{mt}, A_{mt}; μ_{mi}, E_{mi}, G_{mi}, I_{mi}, A_{mi}; and μ_{mb}, E_{mb}, G_{mb}, I_{mb}, A_{mb} are the modifying factor for calculating shear deformation, Young's modulus, the shear modulus, the second-moment of area and the cross-sectional area of the stiffening beams at the top, the intermediate and base level of the walls, respectively, and V_{mb} is the shear force in the bottom stiffening beam.

By equating corresponding terms in eqn (6) to eqn (19) and terms in equations (7a–c) to eqns (20a–c) at level $x = H$, H_1 and 0, the shear forces V_{mt}, V_{mi} and V_{mb} in the stiffening beams are found to be

$$V_{\mathrm{mt}} = S_{\mathrm{mt}}H[q_2(H) - \beta Q(H)] \tag{21a}$$

$$V_{\mathrm{mi}} = S_{\mathrm{mi}}H[q_1(H_1) - \beta Q(H_1)] \tag{21b}$$

$$V_{\mathrm{mb}} = S_{\mathrm{mb}}H[q_0(0) - \beta Q(0)], \tag{21c}$$

where

$$\beta = \gamma \frac{EIb\mu_w}{GAl}$$

and

$$S_{mt} = \frac{E_{mt}I_{mt}h}{EI_bH}\left(1+\frac{12\mu_b EI_b}{GA_b b^2}\right)\left(1+\frac{12\mu_{mt}E_{mt}I_{mt}}{G_{mt}A_{mt}b^2}\right)^{-1} \tag{22a}$$

$$S_{mi} = \frac{E_{mi}I_{mi}h}{EI_bH}\left(1+\frac{12\mu_b EI_b}{GA_b b^2}\right)\left(1+\frac{12\mu_{mi}E_{mi}I_{mi}}{G_{mi}A_{mi}b^2}\right)^{-1} \tag{22b}$$

$$S_{mb} = \frac{E_{mb}I_{mb}h}{EI_bH}\left(1+\frac{12\mu_b EI_b}{GA_b b^2}\right)\left(1+\frac{12\mu_{mb}E_{mb}I_{mb}}{G_{mb}A_{mb}b^2}\right)^{-1}. \tag{22c}$$

The values of B_2, C_2, B_1, C_1, B_0 and C_0 can be determined by considering the boundary conditions. The boundary conditions of the problem are:

(1) the vertical force equilibrium requirement in the walls where the top and the intermediate stiffening beams are located, and at the position of the unit force;

(2) the compatibility requirement for the vertical deflection of the assumed "cut" end of the laminae at the position of the unit lateral force, the intermediate stiffening beam, and the bottom stiffening beam.

These result in:

if $H_1 \leqslant H_p$

$$T_2(H) = V_{mt} \tag{23a}$$

$$T_2(H_p) = T_1(H_p) \tag{23b}$$

$$T_1(H_1) + V_{mi} = T_0(H_1) \tag{23c}$$

$$q_1(H_p) = q_2(H_p) + \beta \tag{23d}$$

$$q_1(H_1) = q_0(H_1) \tag{23e}$$

$$l\theta_0 - \frac{V_{mb}b^3}{12E_{mb}I_{mb}} - \frac{b\mu_{mb}V_{mb}}{G_{mb}A_{mb}} - s = 0 \tag{23f}$$

and if $H_p < H_1$

$$T_2(H) = V_{mt} \tag{24a}$$

$$T_2(H_1) + V_{mi} = T_1(H_1) \tag{24b}$$

$$T_1(H_p) = T_0(H_p) \tag{24c}$$

$$q_1(H_1) = q_2(H_1) \tag{24d}$$

$$q_1(H_p) + \beta = q_0(H_p) \tag{24e}$$

$$l\theta_0 - \frac{V_{mb}b^3}{12E_{mb}I_{mb}} - \frac{b\mu_{mb}V_{mb}}{G_{mb}A_{mb}} - s = 0, \tag{24f}$$

where the base rotation θ_0 and the relative settlement s of the walls at the foundation are given by

$$\theta_0 = \frac{M_e(0) - T_0(0)l - V_{mb}l}{K_r} \tag{25a}$$

$$s = \frac{T_0(0) + V_{mb}}{K_v} \tag{25b}$$

in which K_r and K_v are the rotational and vertical elastic stiffnesses of the foundation, respectively.

Substituting eqns (15), (17), (21) and (25) into eqns (23a–f) and substituting equations (16), (18), (21) and (25) into eqns (24a–f) yields two sets of linear equations, namely:

if $H_1 \leqslant H_p$

$$B_2[\cosh \alpha H + S_{mt}\alpha H \sinh \alpha H] + C_2[\sinh \alpha H + S_{mt}\alpha H \cosh \alpha H] = 0 \tag{26a}$$

$$(B_2 - B_1)\cosh \alpha H_p + (C_2 - C_1)\sinh \alpha H_p = 0 \tag{26b}$$

$$B_1[\cosh \alpha H_1 - S_{mi}\alpha H \sinh \alpha H_1] + C_1[\sinh \alpha H_1 - S_{mi}\alpha H \cosh \alpha H_1]$$

$$- B_0 \cosh \alpha H_1 - C_0 \sinh \alpha H_1 = -\left(\frac{\gamma}{\alpha^2} - \beta\right) S_{mi} H \tag{26c}$$

$$(B_2 - B_1)\sinh \alpha H_p + (C_2 - C_1)\cosh \alpha H_p = -\frac{1}{\alpha}\left(\frac{\gamma}{\alpha^2} - \beta\right) \tag{26d}$$

$$B_1 \sinh \alpha H_1 + C_1 \cosh \alpha H_1 - B_0 \sinh \alpha H_1 - C_0 \cosh \alpha H_1 = 0 \tag{26e}$$

$$\mu_f B_0 - \alpha(1 + \mu_f S_{mb} H)C_0 = -\left(\frac{\gamma}{\alpha^2} - \beta\right)(1 + \mu_f S_{mb} H) + \left(\frac{\lambda_r}{l} - \frac{\gamma}{\alpha^2}\mu_f\right) H_p \tag{26f}$$

and if $H_p < H_1$

$$B_2[\cosh \alpha H + S_{mt}\alpha H \sinh \alpha H] + C_2[\sinh \alpha H + S_{mt}\alpha H \cosh \alpha H] = 0 \tag{27a}$$

$$B_2[\cosh \alpha H_1 - S_{mi}\alpha H \sinh \alpha H_1] + C_2[\sinh \alpha H_1 - S_{mi}\alpha H \cosh \alpha H_1]$$

$$- B_1 \cosh \alpha H_1 - C_1 \sinh \alpha H_1 = -\left(\frac{\gamma}{\alpha^2} - \beta\right) S_{mi} H \tag{27b}$$

$$(B_1 - B_0)\cosh \alpha H_p + (C_1 - C_0)\sinh \alpha H_p = 0 \tag{27c}$$

$$B_2 \sinh \alpha H_1 + C_2 \cosh \alpha H_1 - B_1 \sinh \alpha H_1 - C_1 \cosh \alpha H_1 = 0 \tag{27d}$$

$$(B_1 - B_0)\sinh \alpha H_p + (C_1 - C_0)\cosh \alpha H_p = -\frac{1}{\alpha}\left(\frac{\gamma}{\alpha^2} - \beta\right) \tag{27e}$$

$$\mu_f B_0 - \alpha(1 + \mu_f S_{mb} H)C_0 = -\left(\frac{\gamma}{\alpha^2} - \beta\right)(1 + \mu_f S_{mb} H) + \left(\frac{\lambda_r}{l} - \frac{\gamma}{\alpha^2}\mu_f\right) H_p, \tag{27f}$$

where

$$\mu_f = \lambda_r + \lambda_v, \quad \lambda_r = \frac{12EI_b l^2}{K_r h b^3}, \quad \lambda_v = \frac{12EI_b}{K_v h b^3}.$$

The solutions of the simultaneous equations (26a–f) and (27a–f) give the values of the integration constants B_2, C_2, B_1, C_1, B_0 and C_0.

Having determined the values of B_2, C_2, B_1, C_1, B_0 and C_0, the axial force in the walls can be obtained from eqns (15) and (16). Then, the lateral deflection expressions of the walls due to bending can be derived by integrating eqn (8) twice, i.e.

if $H_1 \leqslant H_p$

$$y_{M2} = \frac{1}{EI}\left[\left(1-\frac{\gamma}{\alpha^2}l\right)(3x-H_p)\frac{H_p^2}{6} - \frac{l}{\alpha^2}(B_2\cosh\alpha x + C_2\sinh\alpha x) + D_2x + F_2)\right]$$

for $H_p < x \leqslant H$ (28a)

$$y_{M1} = \frac{1}{EI}\left[\left(1-\frac{\gamma}{\alpha^2}l\right)(3H_p-x)\frac{x^2}{6} - \frac{l}{\alpha^2}(B_1\cosh\alpha x + C_1\sinh\alpha x) + D_1x + F_1)\right]$$

for $H_1 < x \leqslant H_p$ (28b)

$$y_{M0} = \frac{1}{EI}\left[\left(1-\frac{\gamma}{\alpha^2}l)(3H_p-x\right)\frac{x^2}{6} - \frac{l}{\alpha^2}(B_0\cosh\alpha x + C_0\sinh\alpha x) + D_0x + F_0)\right]$$

for $0 \leqslant x \leqslant H_1$ (28c)

and if $H_p < H_1$

$$y_{M2} = \frac{1}{EI}\left[\left(1-\frac{\gamma}{\alpha^2}l\right)(3x-H_p)\frac{H_p^2}{6} - \frac{l}{\alpha^2}(B_2\cosh\alpha x + C_2\sinh\alpha x) + D_2x + F_2)\right]$$

for $H_p < x \leqslant H$ (29a)

$$y_{M1} = \frac{1}{EI}\left[\left(1-\frac{\gamma}{\alpha^2}l\right)(3x-H_p)\frac{H_p^2}{6} - \frac{l}{\alpha^2}(B_1\cosh\alpha x + C_1\sinh\alpha x) + D_1x + F_1)\right]$$

for $H_1 < x \leqslant H_p$ (29b)

$$y_{M0} = \frac{1}{EI}\left[\left(1-\frac{\gamma}{\alpha^2}l\right)(3H_p-x)\frac{x^2}{6} - \frac{l}{\alpha^2}(B_0\cosh\alpha x + C_0\sinh\alpha x) + D_0x + F_0)\right]$$

for $0 \leqslant x \leqslant H_1$. (29c)

The other integration constants D_2, F_2, D_1, F_1, D_0 and F_0 can be determined by satisfying the following wall deflection compatibility requirements:

if $H_1 \leqslant H_p$

$$y_{M0}(0) = 0 \quad (30a)$$

$$y'_{M0}(0) = \theta_0 \quad (30b)$$

$$y_{M0}(H_1) = y_{M1}(H_1) \quad (30c)$$

$$y'_{M0}(H_1) = y'_{M1}(H_1) \quad (30d)$$

$$y_{M1}(H_p) = y_{M2}(H_p) \quad (30e)$$

$$y'_{M1}(H_p) = y'_{M2}(H_p) \quad (30f)$$

and if $H_p < H_1$

$$y_{M0}(0) = 0 \quad (31a)$$

$$y'_{M0}(0) = \theta_0 \quad (31b)$$

$$y_{M0}(H_p) = y_{M1}(H_p) \quad (31c)$$

$$y'_{M0}(H_p) = y'_{M1}(H_p) \quad (31d)$$

$$y_{M1}(H_1) = y_{M2}(H_1) \quad (31e)$$

$$y'_{M1}(H_1) = y'_{M2}(H_1). \quad (31f)$$

Equations (30) and (31) result in:

if $H_1 \leqslant H_p$

$$D_0 = \frac{l}{\alpha} C_0 + EI\theta_0 \tag{32a}$$

$$F_0 = \frac{l}{\alpha^2} B_0 \tag{32b}$$

$$D_1 = \frac{l}{\alpha} [(B_1 - B_0) \sinh \alpha H_1 + (C_1 - C_0) \cosh \alpha H_1] + D_0 \tag{32c}$$

$$F_1 = \frac{l}{\alpha^2} [(B_1 - B_0) \cosh \alpha H_1 + (C_1 - C_0) \sinh \alpha H_1] + (D_0 - D_1) H_1 + F_0 \tag{32d}$$

$$D_2 = \frac{l}{\alpha} [(B_2 - B_1) \sinh \alpha H_p + (C_2 - C_1) \cosh \alpha H_p] + D_1 \tag{32e}$$

$$F_2 = \frac{l}{\alpha^2} [(B_2 - B_1) \cosh \alpha H_p + (C_2 - C_1) \sinh \alpha H_p] + (D_1 - D_2) H_p + F_1 \tag{32f}$$

and if $H_p < H_1$

$$D_0 = \frac{l}{\alpha} C_0 + EI\theta_0 \tag{33a}$$

$$F_0 = \frac{l}{\alpha^2} B_0 \tag{33b}$$

$$D_1 = \frac{l}{\alpha} [(B_1 - B_0) \sinh \alpha H_p + (C_1 - C_0) \cosh \alpha H_p] + D_0 \tag{33c}$$

$$F_1 = \frac{l}{\alpha^2} [(B_1 - B_0) \cosh \alpha H_p + (C_1 - C_0) \sinh \alpha H_p] + (D_0 - D_1) H_p + F_0 \tag{33d}$$

$$D_2 = \frac{l}{\alpha} [(B_2 - B_1) \sinh \alpha H_1 + (C_2 - C_1) \cosh \alpha H_1] + D_1 \tag{33e}$$

$$F_2 = \frac{l}{\alpha^2} [(B_2 - B_1) \cosh \alpha H_1 + (C_2 - C_1) \sinh \alpha H_1] + (D_1 - D_2) H_1 + F_1. \tag{33f}$$

The lateral deflection of the walls due to shear deformation, y_Q, can be calculated from

$$y_Q = \begin{cases} \dfrac{\mu_w x}{GA} + \dfrac{1}{K_h} & x \leqslant H_p \\ \dfrac{\mu_w H_p}{GA} + \dfrac{1}{K_h} & x > H_p \end{cases}, \tag{34}$$

where K_h is the foundation stiffness in the horizontal direction.

The total lateral deflection of the walls, y, is

$$y = y_M + y_Q. \tag{35}$$

Table 1. Dimensions and properties of example structure†

Wall section:	0.5×8 m	$A_1 = A_2 = 4$ m^2	$I_1 = I_2 = 21.333$ m^4
Coupling beam section:	0.5×0.333 m	$A_b = 0.1667$ m^2	$I_b = 0.001543$ m^4
Stiffening beam section:	0.5×1.0 m	$A_m = 0.5$ m^2	$I_m = 0.04167$ m^4
$H = 60$ m	$h = 3$ m	$b = 2$ m	$l = 10$ m
$E = E_m = 15\times10^6$ kN m^{-2}	$G = G_m = 6\times10^6$ kN m^{-2}		$\rho = 2400$ kg m^{-3}

†I_1, I_2, second moment of area of walls 1 and 2; ρ, density of walls.

NUMERICAL EXAMPLE

A typical coupled shear wall structure reinforced by the stiffening beams positioned at the top and the bottom of the walls and at the middle of the structural height is analysed as an example. The dimensions and relevant properties of the structure are listed in Table 1. Three sets of numerical values for the stiffness properties of foundations are considered:

(1) $K_r = \infty$, $K_v = \infty$, $K_h = \infty$, corresponding to a rigid foundation;

(2) $K_r = 2.72\times10^{10}$ N m rad^{-1}, $K_v = 8.78\times10^8$ N m^{-1}, $K_h = 7.52\times10^8$ N m^{-1}, corresponding to a dense soil foundation;

(3) $K_r = 0.54\times10^{10}$ N m rad^{-1}, $K_v = 1.46\times10^8$ N m^{-1}, $K_h = 1.36\times10^8$ N m^{-1}, corresponding to a soft soil foundation.

The first 10 natural frequency results, obtained by excluding and including the effects of shear deformation, of the structure situated on the above-described foundations are given in Table 2. It is shown that the effects of shear deformation on the fundamental frequencies of stiffened coupled shear walls are slight, especially in soft foundation situations, but the effects of shear deformation on the higher frequencies are significant. In addition, it is clear that the shear deformation of the walls, rather than connecting and stiffening beams, is the dominating influence on the free vibration behaviour of coupled shear wall structures.

To show the effect of the presence of stiffening beams on the natural frequencies of coupled shear walls, a comparison is made in Table 3 on the first 10 frequencies of the example coupled shear walls with various stiff beams. The second set of foundation properties is chosen for this comparison and the cross-section of the stiffening beams considered is the same as listed in Table 1. It is demonstrated that the natural frequencies of coupled shear walls increase with the contribution of stiffening beams, which indicates the improvement in stiffness of coupled shear walls due to the incorporation of stiffening beams.

Table 2. Natural frequencies of the example structure (hertz)

Foundation case	Shear effects considered†	Mode number									
		1	2	3	4	5	6	7	8	9	10
1	N	1.665	7.001	19.19	32.86	53.32	77.61	109.4	141.5	180.9	223.9
	C	1.650	6.957	19.04	32.75	53.19	77.50	109.1	141.4	180.7	223.7
	C+S	1.603	6.884	18.43	32.47	52.85	77.18	108.1	141.0	180.3	223.3
	C+S+W	1.553	6.335	15.40	25.12	36.60	48.35	60.48	71.85	82.82	93.18
2	N	1.055	5.322	12.24	23.55	37.93	58.35	85.69	118.5	153.2	193.6
	C	1.050	5.287	12.18	23.37	37.80	58.21	85.50	118.2	153.0	193.4
	C+S	1.032	5.168	11.98	22.69	37.42	57.94	84.77	116.9	152.3	193.0
	C+S+W	1.018	4.957	11.18	19.70	30.01	41.83	54.54	66.83	78.43	89.34
3	N	0.546	3.342	9.640	22.05	36.71	56.43	84.56	117.5	152.5	192.0
	C	0.545	3.334	9.568	21.84	36.57	56.27	84.37	117.2	152.3	191.8
	C+S	0.542	3.301	9.383	20.95	36.08	56.10	83.63	115.8	151.5	191.6
	C+S+W	0.540	3.254	8.894	18.17	28.95	40.70	53.92	66.30	78.07	88.91

† C, S and W denote shear effects in connecting beams, stiffening beams and structural walls, respectively, and N denotes shear effects ignored.

Table 3. Variation of natural frequencies of the example coupled shear walls (hertz)

Stiffening case†	Mode number									
	1	2	3	4	5	6	7	8	9	10
N	0.921	4.486	10.48	18.53	29.16	41.19	53.68	65.98	77.80	88.93
T	0.944	4.693	10.89	18.95	29.70	41.63	54.14	66.34	78.16	89.21
T + B	0.970	4.923	10.89	18.98	30.00	41.79	54.47	66.50	78.43	89.34
T + B + M	1.018	4.957	11.18	19.70	30.00	41.83	54.54	66.83	78.43	89.34

† T, B and M denote the contribution of the stiffening beams located at the top, the base and the middle level of the structural height, respectively, and N denotes the absence of stiffening beams.

CONCLUSIONS

A hybrid approach allowing for the effects of shear deformation is presented for the free vibration analysis of flexibly based coupled shear walls strengthened by a top and/or a bottom and/or an arbitrary intermediate stiffening beam. This approach, which is based both on the continuous medium approach for the static analysis to obtain the stiffness matrix and on the discrete approach for a dynamic analysis for an equivalent multi-degree-of-freedom system, has the advantage that the advantages of the two types of analytical approaches can be jointly utilized. Since only the basic structural dimensions and relevant properties are involved in the present analysis, the data preparation effort for computing free vibration characteristics of stiffened coupled shear wall structures can be greatly reduced. However, it should be pointed out that the analysis is restricted to cases where EI_i/K_{ri}, i.e. the ratio of the stiffness of the shear walls to that of their foundations, is constant for every pair of shear-wall and its corresponding foundation, for uniform soil conditions.

The analysis of a typical structure demonstrates that the effect of shear deformation on the higher frequencies of stiffened coupled shear walls is significant but is minimal on the structural fundamental frequencies. It is further shown that the free vibration characteristics of stiffened coupled shear wall structures are mainly affected by the shear deformation of the walls rather than that of the coupling or the stiffening beams. In addition, it is anticipated that the introduction of stiffening beams will increase the natural frequencies of coupled shear wall structures.

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